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EXAM ID NUMBER:________

COURSE NUMBER: ___EE 271

PROBLEM: ___1____

EE Qualifying Exam

Control Theory

1. (20pts) Construct a linear system

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

that simultaneously satisfies all of the following requirements:

- 4th order, 1 input, 1 output
- Unstable
- Stabilizable & Detectable
- $\text{rank}\begin{pmatrix} B & AB & A^2B & A^3B \end{pmatrix} = 2$
- $\text{rank}\begin{pmatrix} C \\ CA \\ CA^2 \\ CA^3 \end{pmatrix} = 2$
- $C(sI - A)^{-1}B$ is first order, i.e, can be written as $\frac{\alpha}{s+\beta}$ for some scalar constants α and β.

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EXAM ID NUMBER:________

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PROBLEM: 2

2. (20pts) Consider a time-invariant system

$$\dot{x} = Ax(t), \quad x(0) = x_o$$

The solution is given by the matrix exponential, e^{At}, i.e.,

$$x(t) = e^{At}x(0)$$

Now suppose that A a block upper-triangular matrix, i.e.,

$$A = \begin{pmatrix} A_1 & A_2 \\ 0 & A_4 \end{pmatrix}.$$

(a) Show that the matrix exponential, e^{At} has a similar structure. Specifically, derive an expression for e^{At} in terms of $e^{A_1 t}, e^{A_4 t}$, and A_2.

(b) Now consider the *controlled* case

$$\dot{x} = \begin{pmatrix} A_1 & A_2 \\ 0 & A_4 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u$$

under the special assumption that $A_4 = 0$ and is a scalar (i.e., has dimension 1×1). Under what conditions is the overall system controllable?