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EXAM ID NUMBER:________

COURSE NUMBER: EE 221

PROBLEM: 1

Problem 1 (20 points)

Consider the dielectric slab waveguide shown in the figure. The thickness of slab is t and it extends from $-\infty$ to ∞ in both y and z directions. The TM odd-mode fields in region 1 and region 2 are given below.

Region 1:

$$E_{1z} = -Bv^2 e^{-vx} e^{i\beta z}$$

$$E_{1x} = -Biv\beta e^{-vx} e^{i\beta z}$$

$$H_{1y} = -Bi\omega\varepsilon_1 v e^{-vx} e^{i\beta z}$$

Region 2:

$$E_{2z} = Ak_{2c}^2 \sin(k_{2c}x) e^{i\beta z}$$

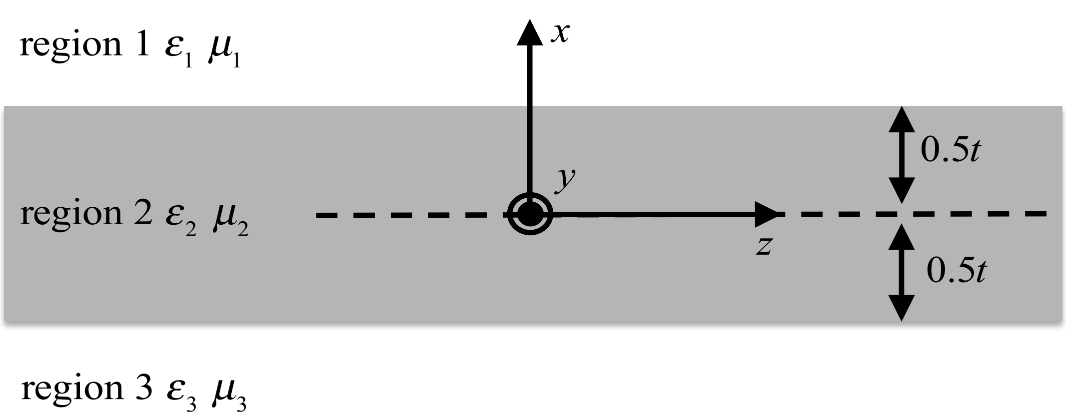
$$E_{2x} = Aik_{2c}\beta \cos(k_{2c}x) e^{i\beta z}$$

$$H_{2y} = Ai\omega\varepsilon_2 k_{2c} \cos(k_{2c}x) e^{i\beta z}$$

Here $k_{2c}^2 = k_2^2 - \beta^2$, $v^2 = \beta^2 - k_1^2$, $\varepsilon_1 = \varepsilon_3$, and $\mu_1 = \mu_2 = \mu_3 = \mu_0$.

(a) Find two equations in v and k_{2c} . (Hint: The other variables in these equations are t , ω , ε_1 , ε_2 , and μ_0) Suggest a graphical method to solve these equations. **(13 points)**

(b) Assuming $\varepsilon_2 = 4\varepsilon_0$ and $\varepsilon_1 = \varepsilon_3 = \varepsilon_0$, what is the minimum thickness t necessary to support the second propagating TM mode? **(7 points)**



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EXAM ID NUMBER:________

COURSE NUMBER: EE 221

PROBLEM: 2

Problem 2 (20 points)

Show that the electric field due to a cylinder of electric current $\mathbf{J} = J_s \hat{a}_z$ (located at $\rho = a$) is

$$\mathbf{E} = \begin{cases} -\hat{a}_z \frac{\pi}{2} \eta_0 k a J_s H_0^{(2)}(ka) J_0(k\rho) & \rho < a \\ -\hat{a}_z \frac{\pi}{2} \eta_0 k a J_s J_0(ka) H_0^{(2)}(k\rho) & \rho > a \end{cases}.$$

Here, $k = \omega\sqrt{\varepsilon_0 \mu_0}$ and $\eta_0 = \sqrt{\mu_0 / \varepsilon_0}$. The cylinder extends from $z = -\infty$ to $z = \infty$ and it is centered at the origin. You may find the following mathematical identities useful:

$$\frac{\partial}{\partial z} J_0(\alpha z) = -\alpha J_1(\alpha z)$$

$$\frac{\partial}{\partial z} H_0^{(2)}(\alpha z) = -\alpha H_1^{(2)}(\alpha z)$$

$$J_0(z) H_1^{(2)}(z) - J_1(z) H_0^{(2)}(z) = \frac{2j}{\pi z}$$