Electrical Engineering – Written PhD Qualifier Exam
Fall 2013

Saturday, September 7th 2013

Room 3119, level 3, Building 1

Please do not write your name on this page or any other page you submit with your work.

Please return your solutions to only 5 questions

Student Identification Number: ___________________________

Optimization

AMCS 212

Problem 1 (20 Points). Consider a set of n points in the plane with coordinates $(x_i, y_i),\ i = 1..n$. We would like to fit an ellipse of the form: $a\,x^2 + b\,y^2 - 1 = 0$ to this given data set. Two different strategies are proposed for generating the unknown coefficients a and b.

Part a (10 Points). In the first strategy, the residual in the fit of a given point is defined to be $r_i = a\,x_i^2 + b\,y_i^2 - 1$, and the problem may be formulated as the minimization of the sum of the squares of the residuals.

- Write down the mathematical optimization problem.
- Describe precisely how you would solve the problem (what is the linear system of equations this is solved? how many iterations are needed?).

Part b (10 Points). In the second strategy we are interested in minimizing the *maximum* residual r_i among all the points.

- Write down the mathematical optimization problem.
- Formulate it as a smooth constrained linear optimization problem.
- Describe, very briefly, how you would solve the problem (what is the linear system of equations that is solved at every iteration?).
- How does the resulting fit obtained with this strategy differ from the fit obtained using the strategy of Part a. Only a brief qualitative answer is needed.

Optimization

AMCS 212

Problem 2 (20 Points) Consider the following optimization problem where $x \in R^n$ and P is a symmetric but not necessarily positive definite matrix.

$$\begin{array}{ll} \text{minimize} & 1/2\, x^t P x + q^t x \\ \text{subject to} & x_i^2 \leq 1, \qquad i = 1..n \end{array}$$

- (5 Points) Form the Lagrangian $L(x, \lambda)$ of the problem.

- (10 Points) Derive the dual problem and express it in the form:

  $$\begin{array}{ll} \text{maximize} & -1/2\, q^t(\ldots)q + (\ldots) \\ \text{subject to} & (\ldots) \end{array}$$

  where (…) denote parts of the dual objective and constraints that you will derive.

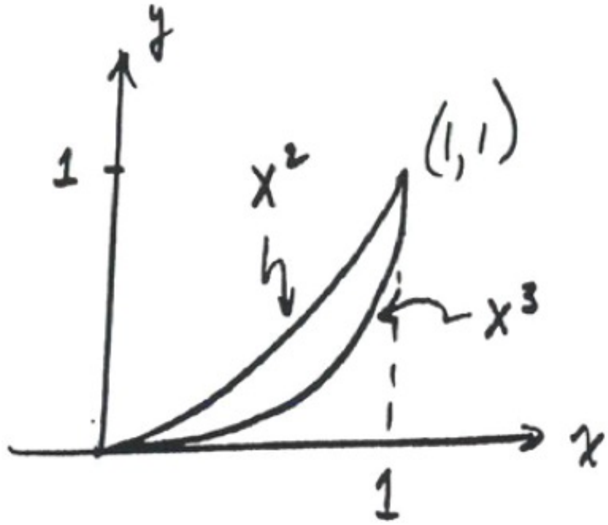
- (5 Points) Comment on the ease/difficulty of solving the dual problem vs solving the primal problem. What can you say about the solution of the dual problem?

Probability and Random Processes

EE 241

Problem 1 (20 Points)

Part 1: (10 points)
Suppose the pair (X, Y) is uniformly distributed over the crescent shown below. Find the correlation coefficient ρ_{xy}.



Part 2: (10 points)
Let $\{X_i\}$ be a sequence of independent and identically distributed (i.i.d.) Normal random variables with zero mean and unit variance. Let

$$S_k = X_1 + X_2 + \cdot + X_k \text{, for } k \geq 1$$

Determine the joint probability density function for S_n and S_m where $1 \leq m < n$.

Probability and Random Processes

EE 241

Problem 2 (20 points)

Let A and B be independent Gaussian random variables with mean values -1 and 0.5, and variances 1 and 1.75, respectively. Define the random process $\{X_t; -\infty < t < \infty\}$ by

$$X_t = A \sin t + B \cos t \qquad -\infty < t < \infty$$

(a) (5 points) Sketch two typical (and different) sample functions of the process.
(b) (5 points) Find the first-order probability density function $f_{X,1}(x;t)$ of the process.
(c) (5 points) Find the mean function $\mu_X(t)$ of the process.
(d) (5 points) Let t_1 and t_2 be two time points with $t_1 < t_2$. Obtain expressions for $E[X(t_1)X(t_2)]$ and $E[(X(t_1) - \mu_X(t_1))(X(t_2) - \mu_X(t_2))]$.
(5 points)

Digital Communications and Coding

EE 242

Problem 1 (20 Points) Ziyad has created a file of size 1M bits that he wants to transmit in 1 msec. The probability of a "1" in this file is 0.45 and of a "0" is 0.55. To do so, he uses the following waveforms

$$s_1(t) = \sqrt{\frac{2}{T}}\, d \cos(2\pi f_c t) \qquad 0 \le t \le T \quad \text{to send 00}$$

$$s_2(t) = \sqrt{\frac{2}{T}}\, d \sin(2\pi f_c t) \qquad 0 \le t \le T \quad \text{to send 11}$$

$$s_3(t) = \sqrt{\frac{2}{T}}\, d \cos(2\pi f_c t + \theta_1) \qquad 0 \le t \le T \quad \text{to send 10}$$

$$s_4(t) = \sqrt{\frac{2}{T}}\, d \cos(2\pi f_c t + \theta_2) \qquad 0 \le t \le T \quad \text{to send 01}$$

where $\theta_1 = \tan^{-1}\frac{1}{2}$ and $\theta_2 = \tan^{-1} 2$.

a. (1 Point) Determine the required bit rate.
b. (1 Point) Determine the symbol rate and the symbol duration T.
c. (2 Points) Determine the average energy of the constellation and the average energy per bit.
d. (4 Points) Determine an orthonormal basis for these sets of waveforms and draw the corresponding constellation points.
e. (5 Points) Determine the decision regions for optimal detection (assume that the symbols are equi-probable).
f. (4 Points) Determine the probability of symbol error. Express your answers in terms of SNR per bit. (In calculating the probability of symbol error, assume that the symbols are equiprobable).
g. (3 Points) It is obvious Ziyad has not taken an course in digital communications which is why his communication system design is lousy. What modifications can you make to the system to make it better?

Digital Communications and Coding

EE 242

Problem 2 (20 Points) Consider the following two constellations to be used in an AWGN channel

(i) The 16-QAM

(ii) The modified QAM (M16-QAM) obtained by moving the corner points in 16-QAM to the I and Q axes.

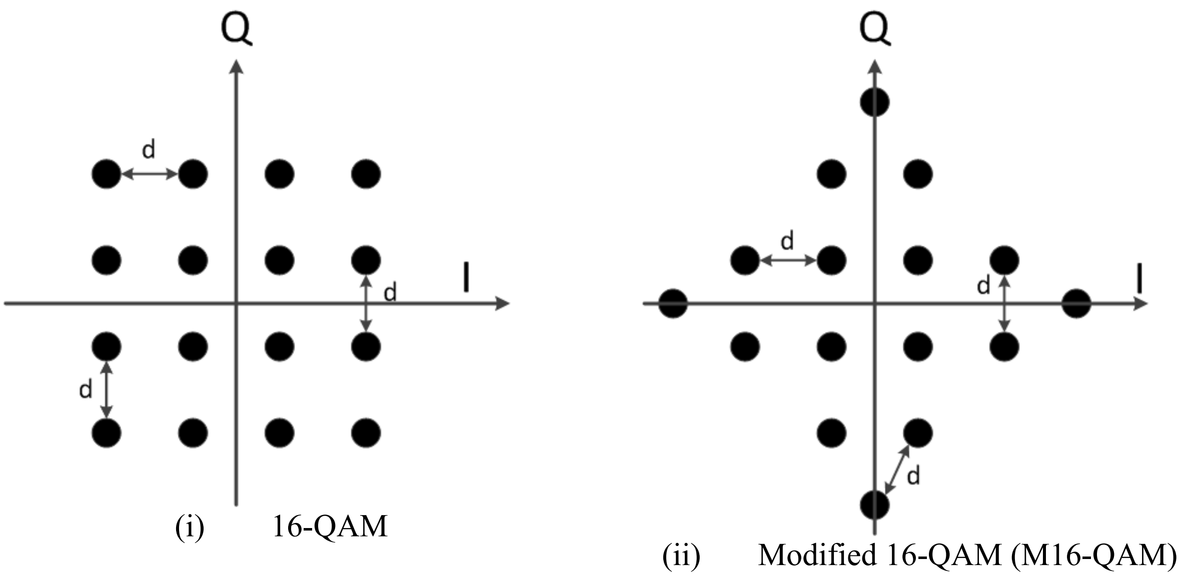


Figure 1: Constellations for Q2

a. (2 Points) Determine the average energy of each constellation.
b. (2 Points) Determine the average number of nearest neighbors for each constellation.
c. (4 Points) Determine the nearest neighbor union bound (approximation) for each constellation. Express your answer in terms of the average energy.
d. (4 Points) Assuming Gray coding, determine the probability of bit error in the high SNR regime. Express your answer in terms of the average energy per bit.
e. (3 Points) Which constellation is more bandwidth efficient? Which one is more energy efficient? Explain your answer.
f. (3 Points) Give two advantages of using rectangular QAM over non-rectangular QAM. Give one advantage of non-rectangular QAM.

Digital Signal Processing

EE 251

Problem 1 (20 Points)

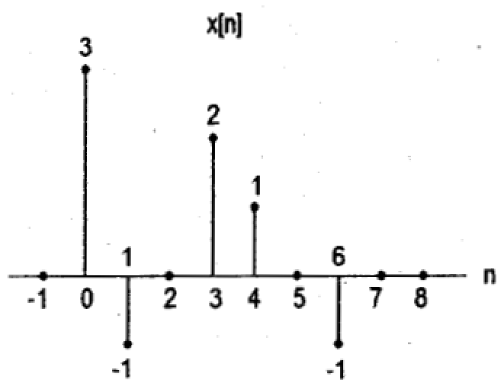
(a) (10 points) How can we compute the N-point DFT of two real-valued sequences using one N-point DFT? That is, in your algorithm, you are allowed one and only one N-point DFT operation. What is required is the DFT of both real-valued sequences.

(b) (10 points) Based on your solution to (a), how can the $2N$-point DFT of a real-valued sequence of $2N$ samples be computed using an N-point DFT one time only?

Digital Signal Processing

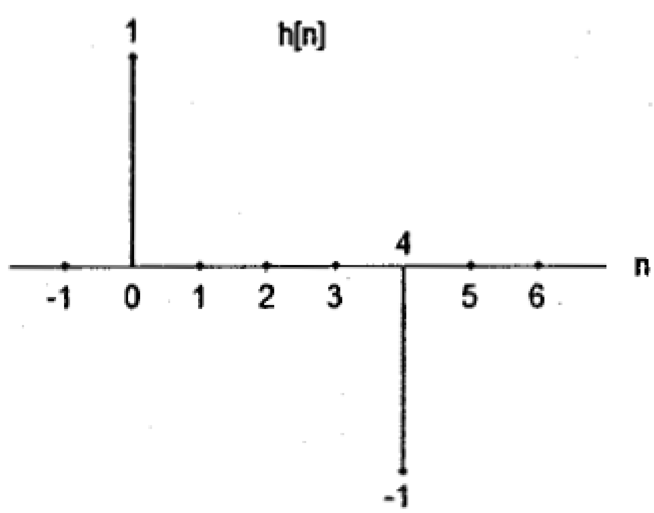
EE 251

Problem 2 (20 Points)



(a) (10 points) Let $X(f)$ be the spectrum of the discrete-time signal $x(n)$ shown above, $T_s = 1$. If four samples are taken from $X(f)$ at frequencies $\frac{kf_S}{4}$, *k* = 0,1,2,3, and then these four samples are inverse Fourier transformed. What is the resulting signal in the time domain at *n*=0,1,2,3?

(b) (10 points) Let $X(k)$ be the 8-point DFT of $x(n)$ and let $H(k)$ be the 8-point DFT of the impulse response $h(n)$ shown in the figure below. Define $Y(k) = X(k)H(k)$ for $0 \leq k \leq 7$. Find $y(n)$, the 8-point IDFT of $Y(k)$.



Communication Networks

EE 262

Problem 1 (20 Points)

Suppose that the time between requests to a web server (in milliseconds) is exponentially distributed with $P(X>10)=0.8$.

a) (4 Points) Find the exponential distribution parameter λ.

b) (4 Points) Give the mean of the time between requests.

c) (4 Points) Assume that the server has been waiting for 8 msec for the next request, what is the probability that the server waits more than 2 extra msec.

d) (2 Points) What is the probability of having two requests arriving in the same time?

e) (2 Points) Suppose that X and Y have exponential distributions with parameters a and b, respectively, and are independent. Prove that $P(X<Y) = a/a+b$.

Communication Networks

EE 262

Problem 2 (20 Points)

Consider an optical switch that is connected with a fiber optic link with m channels. At certain time t, the switch attempts to schedule a packet and found that the m channels are busy and there exist n number of packets being transmitted prior this packet. Assume there is a sufficient buffering for this packet, the packets are served in FCFS fashion, and no new requests for service are permitted after t. The channel service times are mutually independent, identical, exponentially distributed random variables, each with mean duration $1/\mu$.

a) (4 Points) Find the expected length of time the packet spends waiting for the channel in the queue.

b) (4 Points) Find the expected length of time from the arrival of the packet at time t until the system becomes completely empty

c) (4 Points) Let X be the order of completion of service of packet A: that is, X = k if A is the k^{th} packet to complete service after $t = 0$. Find $P[X = k]$ ($k = 1,2, \ldots, m + n + 1$).

d) (4 Points) Find the probability that the packet is scheduled before the one immediately ahead of it in the queue.

e) (4 Points) Let w be the amount of time packet waits for service. Find $P\left[w > x\right]$.

Control Theory

EE 271 + EE 271 B

Problem 1 (20 Points)

The dynamics of an inverted pendulum is $J\ddot{\theta} = -mglsin(\theta) - \mu\dot{\theta} + u$, where u is an external torque input, J is the moment of inertia, m is the mass of the pendulum, l is the position of the center of gravity and g is the acceleration of gravity. μ is a friction coefficient.

1. (4 Points) Write the nonlinear state space equation of the system, in standard form

2. (3 Points) What equilibrium torque input must be applied to stabilize the pendulum at $\theta = \pi$?

3. (5 Points) Linearize the system around the equilibrium position $\theta = \pi$, and write the dynamical equations of the linearized system in state space form

4. (3 Points) Is the linearized system stable? What can you conclude for the nonlinear system?

5. (3 Points) Is the linearized system controllable?

6. (2 Points) True or false: if a linearized system (around a point) is controllable, then the original nonlinear system is also controllable (completely and globally). If true, show it, if false find a counterexample.

Control Theory

EE 271 + EE 271 B

Problem 2 (20 Points)

Part 1:

We consider a linear system of the form $\dot{x} = Ax + Bu$ and $y = Cx$.

1. (3 Points) Give the criterion on A,B and C for the system to be completely observable or completely controllable.

We now assume that $A = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and $C = \begin{pmatrix} 0 & 1 \end{pmatrix}$.

2. (3 Points) Is this system observable?

3. (5 Points) Design a reduced order observer for this system, with one eigenvalue at -2. Explain how you can infer the value of both states from the observer dynamical equations.

Part 2:

4. (3 Points) Write the solution x(t) to this system as a function of the initial condition x(0) and of the control input u.

5. (3 Points) Recall the definition of exponential stability of the system, when no control input is present. How do you check exponential stability for a linear system in practice?

6. (3 Points) Using the above questions, show that the system is bounded input-bounded state stable (i.e. the norm of the state is bounded provided that the norm of the input is bounded) when the matrix A is exponentially stable (do not use the eigenvalues criterion). Hint: if A is exponentially stable, then $\|e^{At}\| \leq Me^{-\propto t}$ for some strictly positive α.